PROBLEM OF NUMERICAL MODELING OF THE OPERATION OF A LOGARITHMIC ELECTRON MULTIPLIER

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The concept of a "logarithmic electron multiplier," in particular, a logarithmic photoelectron multiplier (PEM), first appeared in [1, 2] in connection with the need to record signals of an a priori unknown power and duration in connection with laser ranging of the atmosphere and ocean with the aim of determining their optical characteristics as well as in connection with the recording of the radiation accompanying some rapidly occurring processes. The phenomenon of logarithmic conversion of the density of an electron stream by the field of an intrinsic space charge lies at the basis of operation of a logarithmic electron multiplier. The complexity of the indicated phenomenon requires special methods of investigation. The physical formulation of the problem reduces to the solution of a time-dependent system of equations: those of Poisson, continuity, and motion in a three-dimensional region which is the collector unit of the electron multiplier with account taken of the self-consistency of the boundary conditions and the distribution function of electrons over the initial velocity vector. As a result it is necessary to obtain the corresponding current density function of dynode electrons as a function of the time. The solution of this problem in its complete formulation does not appear possible due to its categorical mathematical complexity. Therefore, the method of mathematical modeling of the physical phenomenon in question was selected, and numerical experiments were conducted. The main goal for conducting the numerical experiments consisted of obtaining the impulse parameters of the logarithmic phenomenon as well as refining the differential and difference implementations of the original physical model by comparing the calculated characteristics with those obtained as a result of a physical experiment. The time-independent operation of a PEM has been previously investigated [4], and the results of physical experiments have been obtained for the impulse regime [3]. Therefore if the qualitative conclusions, and primarily the fundamental possibility of logarithmic conversion of the density of an electron stream by the field of an intrinsic space charge, have been confirmed, the quantitative characteristics of the physical process have not been completely discussed. For a complete understanding of the necessity of conducting a computational experiment one should note the advantage of the latter in comparison with the possibilities of a physical experiment. At present the response speed of a logarithmic PEM is estimated from the change in the duration of the output pulse at a level specified with respect to its maximum value upon a successive decrease in the value of the output light pulse. The existing sources of nanosecond light pulses do not possess the necessary degree of stability, which leads to distortion of the shape and duration of the input signal and inadequacy of the conditions for making measurements. A computational experiment is devoid of the indicated shortcomings, since one can specify the shape and duration of the input pulses to be constant in it. However, since the problem cannot be solved in its complete formulation, a number of physical and mathematical simplifications are necessary for conducting a computational experiment.

The actual physical region in which logarithmic conversion of the density of an electron stream occurs is the drift interval of an electron multiplier of the "louvered" type. Its construction and the equipotential lines of the field [5] are presented in Fig. 1 (L denotes a dynode of the "louvered" type, S denotes a screen grid, T denotes the possible trajectories of the electrons). The region in Fig. 1 consists of two subregions: a — with a complex field between the vanes of the "louver," and b — the fields of a flat diode. The process of formation of secondary and reflected electrons and the formation of the stream of electrons onto the next diode occur in a. The region a is an emitter of electrons for region b (B and C are the boundaries of region b). We shall assume that the emitting surface coincides with a practically flat equipotential surface, for example, B. We shall restrict ourselves to consideration of processes of the passage of a stream of electrons into b. Electrons emitted into b from the surface B possess some distribution over the velocity vector $f(v)$.

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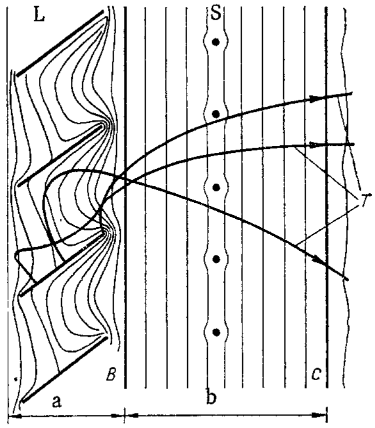

Fig. 1

This distribution is the result of the conversion of the initial distribution of secondary and reflected electrons in the electric field between vanes of the louver in region a. Up to now there has been no general analytic form of the distribution of secondary and reflected electrons over the initial velocity vector. However, analysis of numerous experimental investigations [6] has shown the presence in the distribution of an exponential factor of the type $\exp(-v^2/w)$. As the electrons pass between the vanes of the "louver", the exponential nature of the distribution is preserved both in accelerating and retarding fields [7]. At the same time the angular distribution is stretched out along the normal to the plane B by virtue of the electron-optical characteristics of the diode system [5].

Thus the problem of the passage of a time-varying stream of electrons in a plane diode has been solved on the assumption that the electrons leaving the emitter B have an exponential distribution in the velocity component normal to B.

We shall consider the system of equations

$$\Delta U(x, t) = -4\pi\rho(x, t), \; m\partial v/\partial t + e\partial U/\partial x = 0, \; \partial\rho(x, t)/\partial t + \operatorname{div} j = 0 \tag{1}$$

with the following boundary and initial conditions:

$$U(0, 0) = 0, \; U(x_c, 0) = \varphi_c, \; f(0, 0, v) = (mv^2/2w)\exp(-mv^2/2w), \quad j(0, t) = F(t), \; \rho(x, 0) = 0,$$

where the emitter B is located at the origin of the x axis and has a potential equal to zero, x_c and φ_c are the coordinate and potential of the collector, $f(0, 0, v)$ is the distribution function of electrons over velocity on the emitter at $t = 0$, $F(t)$ is the shape of the input signal, and $\rho(x, 0)$ is the density of the space charge at $t = 0$. The problem described by the system of equations (1) is time-dependent and self-consistent, since the process of passage of a stream of electrons develops in time and the potentials induced by the electrons can alter the boundary conditions.

We shall use for the solution of the system of equations (1) the method of large particles (MLP), which has been described in detail in [8] for time-dependent problems of aero- and hydrodynamics and some applied problems. We shall modify the MLP with application to our model. We shall devote most of our attention to the temporal aspects of the processes of formation and dissipation of the space charge upon the passage of pulsed electron streams.

We shall construct a discrete model of the problem (1). To this end we shall introduce the coordinate system (x, v), where x is the spatial coordinate and v is the velocity of the electrons. We shall consider an Eulerian grid with the spacing Δx in x and Δv in v. Each cell of the grid is specified by two indices (i, k), and a node of the grid is specified as $(i \pm 1/2, k \pm 1/2)$, where $i = 1, \ldots, N$ and $k = 1, \ldots, M$. We shall write the system of difference equations for the MLP

$$\begin{gathered} \frac{U_{i+1}^n - 2U_i^n + U_{i-1}^n}{\Delta x^2} = -4\pi e \sum_{k=1}^{M} f(x_i, t^n, v_k)\,\Delta v, \\ \ldots, \quad i = 1, \ldots, N, \\ \frac{v_k^{n+1} - v_k^n}{\Delta t} + \frac{e}{m}\frac{U_{i+\frac{1}{2}}^n - U_{i-\frac{1}{2}}^n}{\Delta x} = 0, \\ \ldots, \quad k = 1, \ldots, M, \end{gathered} \tag{2}$$

$$\frac{e}{\Delta t}\left(\sum_{k=1}^{M} f(x_i, t^{n+1}, v_k)\Delta v - \sum_{k=1}^{M} f(x_i, t^n, v_k)\Delta v\right) + \frac{\Delta M^n_{i+\frac{1}{2}} - \Delta M^n_{i-\frac{1}{2}}}{\Delta x} = 0,$$
$$\ldots, \quad i = 1, \ldots, N,$$

where Δt is the step in time, $f(x_i, t^n, v_k)$ is a discrete representation of the distribution function for the cell (i, k) up to time t^n, and $\Delta M^n_{i+1/2} = \rho^n_{i+1/2} v^n_{i+1/2}$ determines the transfer of charge and mass through the boundary of the Eulerian grid.

Thus the LP corresponds to the layer Δx and contains M groups of electrons with the velocity v_k, $k = 1, \ldots, M$.

We shall assume that each LP is represented not by a layer Δx but by an infinitely thin plane having charge q_k, mass m_k, and velocity v_k; the ratio q_k/m_k is equal to the ratio e/m for the electron, i.e., we shall assume that electrons with velocity v_k in the layer Δx are concentrated in a single plane — in the plane of the LP. Then a stream of electrons in a plane diode will be represented by a set of moving charged LP planes. It is possible to obtain for such a stream at each time t^n under some conditions the distribution of the potential in analytic form and thereby, omitting the Eulerian stage of the calculation, proceed to the calculation of the characteristics of the field in moving Lagrangian coordinates associated with the LP. The distribution of the potential inside the diode will depend on the distribution of the charge density over the LP. For a distribution of the charge density over LP in the form of an axisymmetric Gaussian function $\sigma(r) = \sigma_0 \exp(-r^2/b^2)$ the potential of the field near the x axis of a plane axisymmetric diode in any plane with the coordinate x is equal to

$$U^n(x) = 2\pi b \sum_{l=1}^{L} \sigma_{0l} \exp\left[\frac{(x - x_l^n)^2}{b^2}\right]\left(\frac{\sqrt{\pi}}{2} \int_0^{(x-x_l)/b} - e^{-z^2} dz\right), \tag{3}$$

where x_l^n is the coordinate of the l-th LP at time t^n, σ_{0l} is the charge density at the center of the l-th LP on the x axis, and L is the number of all LP located inside the plane diode up to time t^n.

Since the component of the electron velocity in the direction of the x axis has been taken into account, the parallel representation of the field (3) does not contradict the initial assumptions.

Calculation of the transfer of charge and mass inside the diode is associated with discrete specification of the current density function on the emitter $j(0, t) \simeq F(t)$; it is convenient to represent $j(0, t)$ as a piecewise-constant function $F(t)$ with a spacing δt and N_1 steps. The charge $Q_{\delta t}$ introduced by electrons from the emitter in a time δt is equal to $Q_{\delta t} = \sum_{k=1}^{M_1} q_k$, i.e., the total charge of all the LP which leave the emitter in a time δt, and the mass is equal to the total mass $M_{\delta t} = \sum_{k=1}^{M_1} m_k$. The quantities q_k and m_k are determined by a discrete representation of the distribution function. In the calculations the distribution of electrons over velocity was specified as a piecewise-constant function with step δv and M_1 steps in the range of initial electron energies of 0-50 eV.

In this scheme the conservation laws do not require additional control, since the configuration of the LP is preserved upon a transition from temporal layer t^n to t^{n+1}.

Self-consistency of the problem (1) is manifested in the change of the boundary conditions under the action of induced charges on the cathode and anode as an electron stream passes. Taking account of the changing boundary conditions is accomplished by addition of the term $\varphi(x, t^n)$ to the expression (3), which represents the solution of the Laplace equation $\Delta\varphi = 0$ in a plane diode under the following boundary conditions:

$$\varphi(0, t^n) = \varphi(0, 0) + U(0, t^n), \quad \varphi(x_c, t^n) = \varphi(x_c, 0) + U(x_c, t^n).$$

As a result the scheme for solution of the system (2) is modified. The system of grid equations for the potential is replaced by the expression (3) with the additional term $\varphi(x, t^n)$, which takes account of the self-consistency of the problem. The intermediate quantities $\tilde{v}_k^n$, $\tilde{x}_k^n$, and $\tilde{U}^n(x)$ are first calculated upon the integration of the equations of motion, and then the transition to v_k^{n+1}, x_k^{n+1}, and $U^{n+1}(x)$, i.e., to the temporal layer $t^{n+1} = t^n + \Delta t$, is carried out with the corrections taken into account. A system of grid equations of conti-

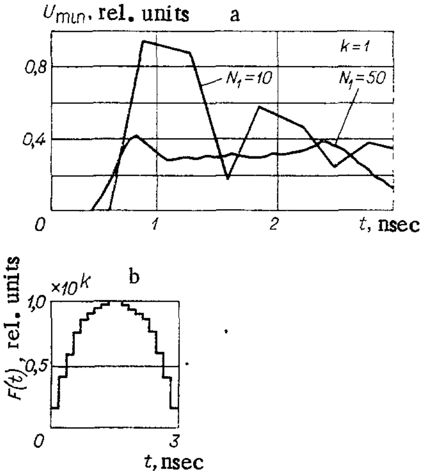

Fig. 2

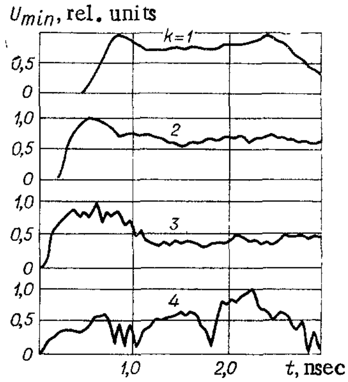

Fig. 3

nuity is necessary for investigations of the behavior of the distribution function of electrons in the process of passage of a stream, the specification of the input signal function, and obtaining the output signal function. Since the dynamics of the distribution function of electrons over velocity or energy in the field of a space charge is not being investigated in this paper, we shall restrict ourselves to consideration of the system of continuity equations near the coordinates of 0 and x_c, i.e., to the specification of F(t) and obtaining a discrete representation of $F_1(t)$ of the output signal.

The stability of the computational scheme has been checked in the course of conducting the numerical experiments. To this end the behavior of the potential barrier function $U(x_{min}, t)$ was controlled, and physically unreal oscillations of $U(x_{min}, t)$ were isolated. Two kinds of behavior of the function $U(x_{min}, t)$ are given in Fig. 2a for a signal whose shape is shown in Fig. 2b. For $N_1 = 50$ the scheme becomes stable. However, the question of the stability of the scheme is complicated by the necessity of modeling the passage of signals with an appreciable (in the range of 3-5 orders of magnitude) varying amplitude. The functions $U(x_{min}, t)$ for the signal of Fig. 2b, k = 1, 2, 3, 4, $N_1 = 50$, and $M_1 = 6$, are shown in Fig. 3. For $k \geqslant 4$ the scheme becomes unstable, and it is necessary to increase the parameter N_1. In this case the time of the calculations may turn out to be unjustifiably great in calculations on a computer of average capacity.

In order to check the reliability of the mathematical model which has been developed, a comparison is made of the results of the computational and physical experiments. The numerical experiment was carried out on a BÉSM-6 computer. The passage of a signal F(t) (Fig. 4a) was modeled with the discretization parameters $N_1 = 600$ and $M_1 = 10$. The corresponding dis-

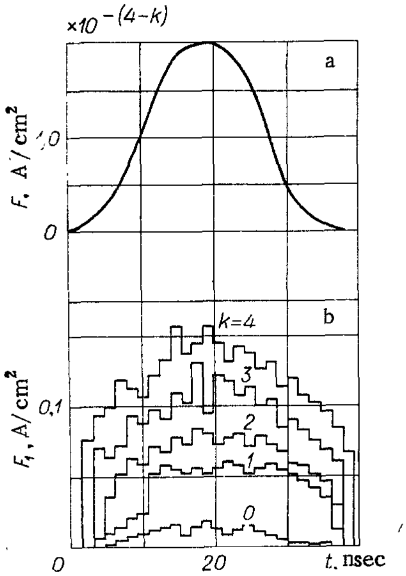

Fig. 4

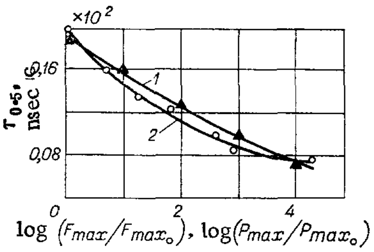

Fig. 5

crete functions of the output signal $F_1(t)$ are shown in Fig. 4b. With the indicated discretization parameters the computational process is stable. The dependence of the halfwidth $\tau_{0.5}$ of the output signal as a function of the maximum value of the input signal F_{max} is constructed from the functions $F_1(t)$ (Fig. 5, curve 1). A similar dependence (Fig. 5, curve 2) was obtained in the physical experiment of [9], in which an FÉU-97 was irradiated by short pulses of laser radiation. The radiation power P was varied with neutral light filters in the 10^5 range. The output signals from the photomultiplier were recorded with a high-speed oscillograph. As follows from Fig. 5, satisfactory agreement of the results is observed.

The developed modification of the method of large particles adequately describes the dynamic phenomena of the passage of streams of electrons in electron multipliers. Nonlinear effects of the interaction of the streams and their influence on the characteristics of the devices are taken into account. It is also possible with the help of the method of large particles to investigate a number of physical characteristics of the process of passage of dense streams of particles with account taken of the space charge phenomenon, namely: the dynamics of the distribution function of the potential between the dynodes, and the dynamics of formation of the output signal on the collector of the multiplier.

The developed modification of the method of large particles can be used to construct special kinds of electron multipliers having a nonlinear calibration characteristic which is realizable with the help of the space charge phenomenon.

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